Index of Nomenclature for Stable Inversion of Piecewise Affine Systems

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1 Introduction

This document provides an index of the nomenclature used in

> I. A. Spiegel, N. Strijbosch, R. de Rozario, T. Oomen, and K. Barton, "Stable Inversion of Piecewise Affine Systems with Application to Feedforward and Iterative Learning Control," *IEEE Transactions on Automatic Control*, Conditionally accepted, 2023.

The article defines all nomenclature as it appears to facilitate a continuous reading experience, and the majority of the notation is standard across the control theory literature. However, piecewise affine systems, nonlinear model inversion, and iterative learning control are all often notationally intensive subjects. Thus, an exhaustive index of abbreviations and notation may create additional convenience.

For mathematical notation (the chief focus of this document), references to where in the article the notation is defined are given. Exceptions are made for extraordinarily common symbols, for which external references are given. External reference is also given when the article gives an external reference.

2 Nomenclature

Enumerations

(A#) Assumption Number #

(C#) Contribution Number #

Initialisms and PWA System Definition Vocabulary

convex polytope an intersection of half spaces

ILC Iterative Learning Control

ILILC Invert-Linearize Iterative Learning Control

location a region of the PWA system state space sharing the same affine time-varying dynamics; the union of all locations is equal to $\mathbb{R}^{n_x}$

LTI Linear Time-Invariant

NMP Non-minimum Phase; commonly used of a system to mean its inverse is unstable

NRMSE Normalized Root Mean Square Error

PD Proportional-Derivative

PEM Peak Error Magnitude

polytope signature vector a unique binary vector corresponding to a particular convex polytope; $\delta_k = H(Px_k - \beta)$ returns the signature if and only if x_k lies in the corresponding polytope

PWA Piecewise Affine

relative degree for a discrete-time system, the number of time steps after k_0 before an input applied at time k_0 influences the output

RMS Root Mean Square

s.t. "such that"

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Mathematical Notation

Note for state space matrices: each state space system matrix is defined individually below, but they can also be grouped by font. Italic refers to a particular location's matrix, upright bold is a PWA system's overall/active matrix, calligraphic font is for output preview, an overline indicates the matrix belongs to the *inverse* of a PWA system, a tilde is for a similarity transformed inverse, and hats ˆ mark variables of known models in the validation example to distinguish them from the unknowable true system.

$0_{\circ\times\bullet}$ ∘-by-• zero matrix, Eq. (49) : Assumption (A9)

$\forall$ universal quantification, "for all" [1]

$\exists$ existential quantification, "there exists" [1]

$\neg$ logical negation, "not" [1]

$\wedge$ logical conjunction, "and" [1]

$\iff$ logical equivalence, "if and only if" [1]

$\implies$ logical implication, "implies" [1]

$[\![\circ, \bullet]\!]$ a closed interval of integers from ∘ to • Definition 1

$\|\bullet\|$ norm for vector •, corresponding operator norm for matrix • : Eq. (47) : Assumption (A8)

$|\bullet|$ cardinality of •, when • is a set [2] : Definition 1 absolute value of •, when • is a scalar [3]

$\emptyset$ empty set [4]

$\circ \in \bullet$ set membership, ∘ is an element of • [5]

$\circ \subseteq \bullet$ subset, ∘ is a subset of • [5]

$\frac{\partial \circ}{\partial \bullet}$ partial derivative of ∘ with respect to • in numerator layout (Jacobian) : Eq. (68)

$\prod_{m=\bullet}^{\circ}$ ordered product of a sequence from ∘ on the left to • on the right : Lemma 2

$\mathbf{A}_k$ active state matrix of PWA system at time k $= \sum_{q=1}^{|Q|} A_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_x \times n_x}$: Eq. (2-5)

$A_{q,k}$ state matrix of location q at time k $\in \mathbb{R}^{n_x \times n_x}$: Eq. (1) : Definition 1

$\overline{\mathbf{A}}_k$ active state matrix of a PWA inverse at time k $\in \mathbb{R}^{n_x \times n_x}$: Eq. (14), (28) : Theorems 1 & 2

$\tilde{\mathbf{A}}_k^{\mathscr{s}}$ active state mat. of a PWA inverse's stable modes $\in \mathbb{R}^{n_{\mathscr{s}} \times n_{\mathscr{s}}}$: Eq. (49) : Assumption (A9)

$\tilde{\mathbf{A}}_k^{\mathscr{u}}$ active state mat., PWA inverse's unstable modes $\in \mathbb{R}^{n_{\mathscr{u}} \times n_{\mathscr{u}}}$: Eq. (49) : Assumption (A9)

$\tilde{A}_{q,k}^{\mathscr{u}}$ state mat. of location q of inv.'s unstable modes $\in \mathbb{R}^{n_{\mathscr{u}} \times n_x}$: Eq. (59)

$\hat{\mathbf{A}}_k$ active state mat., known monolithic printer model $= \begin{bmatrix} \hat{A}^P - \hat{B}^P \hat{\mathbf{D}}_k^C \hat{C}^P & \hat{B}^P \hat{\mathbf{C}}_k^C \\ -\hat{\mathbf{B}}_k^C \hat{C}^P & \hat{\mathbf{A}}_k^C \end{bmatrix}$: Eq. (62) : Table I

$\mathbf{A}_k^C$ state mat., true PWA example controller model $= \begin{bmatrix} 0 & 1 & 0 \\ -a_2 & -a_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$: Figure 2 : Table I

$\hat{\mathbf{A}}_k^C$ state mat., known PWA example controller model $= \begin{bmatrix} 0 & 1 & 0 \\ -a_2 & -a_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$: Eq. (60) : Table I

A^P state mat. of unknown true LTI example plant $= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0.923 & 9.32\text{E}{-4} & 0.0774 & 6.79\text{E}{-5} \\ 0 & -150.5 & 0.841 & 150.5 & 0.156 \\ 0 & 9.08\text{E}{-4} & 7.96\text{E}{-7} & 0.999 & 9.99\text{E}{-4} \\ 0 & 1.77 & 1.86\text{E}{-3} & -1.766 & 0.998 \end{bmatrix}$ Figure 2 : Table I

$\hat{A}^P$ state mat. of the known LTI example plant model $= \begin{bmatrix} 0.668 & 1 & 0 & 0 \\ -0.377 & 0.6683 & 1.90 & -6.01 \\ 0 & 0 & 0.989 & -10.14 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Section V-A : Table I

a_1, a_2 lowpass filter transfer function denominator coefficients : $\in \mathbb{R}$: Eq. (60,75) : Table I

$\mathbb{B}$ boolean domain [6]

$\mathbf{B}_k$ active input matrix of PWA system at time k $= \sum_{q=1}^{|Q|} B_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_x \times n_u}$: Eq. (2-5)

$B_{q,k}$ input matrix of location q at time k $\in \mathbb{R}^{n_x \times n_u}$: Eq. (1) : Definition 1

$\overline{\mathbf{B}}_k$ active input matrix of a PWA inverse at time k $\in \mathbb{R}^{n_x \times n_u}$: Eq. (14), (28) : Theorems 1 & 2

$\tilde{\mathbf{B}}_k^{\mathscr{s}}$ active input mat. of PWA inverse's stable modes $= \mathcal{I}^{\mathscr{s}} V \overline{\mathbf{B}}_k \in \mathbb{R}^{n_{\mathscr{s}} \times n_u}$: Eq. (53)

$\tilde{\mathbf{B}}_k^{\mathscr{u}}$ active input mat. of PWA inv.'s unstable modes $= \mathcal{I}^{\mathscr{u}} V \overline{\mathbf{B}}_k \in \mathbb{R}^{n_{\mathscr{u}} \times n_u}$: Eq. (53)

$\tilde{B}_{q,k}^{\mathscr{u}}$ input mat. of location q of inv.'s unstable modes $\in \mathbb{R}^{n_{\mathscr{u}} \times n_u}$: Eq. (59)

$\hat{\mathbf{B}}_k$ input matrix of known monolithic example model $= \begin{bmatrix} \hat{B}^P \\ 0_{n_{\hat{x}^C} \times 1} \end{bmatrix}$: Eq. (62) : Table I

$\mathbf{B}_k^C$ input matrix of true PWA example controller $= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$: Figure 2 : Table I

$\hat{\mathbf{B}}^C_k$ input matrix of known PWA example controller $= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$: Eq. (60) : Table I

B^P input mat. of unknown true LTI example plant $= \begin{bmatrix} 0 & 4.79\text{E}{-4} & 0.932 & 2.42\text{E}{-7} & 7.96\text{E}{-4} \end{bmatrix}^T$ Figure 2 : Table I

$\hat{B}^P$ input mat. of the known LTI example plant model $\begin{bmatrix} 0 & 2.59\text{E}{-4} & 4.37\text{E}{-4} & 1.38\text{E}{-3} \end{bmatrix}^T$ Section V-A : Table I

b lowpass filter transfer function numerator coefficient : $\in \mathbb{R}$: Eq. (60,75) : Table I

$\mathbf{C}_k$ active output matrix of PWA system at time k $= \sum_{q=1}^{|Q|} C_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_y \times n_x}$: Eq. (2-5)

$C_{q,k}$ output matrix of location q at time k $\in \mathbb{R}^{n_y \times n_x}$: Eq. (1) : Definition 1

$\mathcal{C}_k$ output matrix of a $\mu_g \geq 1$ PWA output preview $= \mathbf{C}_{k+\mu_g} \left(\prod_{m=0}^{\mu_g - 1} \mathbf{A}_{k+m} \right)$: Eq. (16) : Lemma 2

$\overline{\mathbf{C}}_k$ active output matrix of a PWA inverse at time k $\in \mathbb{R}^{n_y \times n_x}$: Eq. (14), (28) : Theorems 1 & 2

$\hat{\mathbf{C}}_k$ output matrix, known monolithic example model $= \begin{bmatrix} \hat{C}^P & 0_{1 \times n_{\hat{x}^C}} \end{bmatrix}$: Eq. (62) : Table I

$\mathbf{C}^C_k$ output mat. of true PWA example controller $= -b \begin{bmatrix} \frac{K_d(1+a_2)}{T_s} + K_p a_2 \\ \frac{K_d a_1}{T_s} + K_p(a_1 - 1) \\ 0 \end{bmatrix}^T$: Figure 2 : Table I

$\hat{\mathbf{C}}^C_k$ output mat. of known PWA example controller $= -b \begin{bmatrix} \frac{K_d(1+a_2)}{T_s} + K_p a_2 \\ \frac{K_d a_1}{T_s} + K_p(a_1 - 1) \\ 0 \end{bmatrix}^T$: Eq. (60) : Table I

C^P output mat. of unknown true LTI example plant $= \begin{bmatrix} 1 & 0_{1\times 4} \end{bmatrix}$: Figure 2 : Table I

$\hat{C}^P$ output mat. of known LTI example plant model $= -\begin{bmatrix} 4.72\text{E}4 & 9.21\text{E}4 & 0_{1\times 2} \end{bmatrix}$ Section V-A : Table I

c_k total voltage applied to positioning system motor $= y^C_k + u_k \in \mathbb{R}$: Figure 2 : Section V-A

$C^{LP}(z)$ lowpass filter transfer func. in the example system's feedback controller : $\in \mathbb{C} \to \mathbb{C}$: Eq. (75)

$C^{PD}(z)$ PD controller transfer func. in the example system's feedback controller : $\in \mathbb{C} \to \mathbb{C}$: Eq. (76)

$\mathbf{D}_k$ active passthrough mat. of PWA sys. at time k $= \sum_{q=1}^{|Q|} D_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_y \times n_u}$: Eq. (2-5)

$D_{q,k}$ passthrough matrix of location q at time k $\in \mathbb{R}^{n_y \times n_y}$: Eq. (1) : Definition 1

$\mathcal{D}_k$ passthrough mat. of $\mu_g \geq 1$ PWA output preview $= \mathbf{C}_{k+\mu_g} \left(\prod_{m=1}^{\mu_g - 1} \mathbf{A}_{k+m} \right) \mathbf{B}_k$: Eq. (16) : Lemma 2

$\overline{\mathbf{D}}_k$ active passthrough mat. of PWA inv. at time k $\in \mathbb{R}^{n_y \times n_u}$: Eq. (14), (28) : Theorems 1 & 2

$\hat{\mathbf{D}}_k$ passthrough matrix of known monolithic example $= 0$: Eq. (62) : Table I

$\mathbf{D}^C_k$ passthrough mat., true PWA example controller $= b\left(K_p + \frac{K_d}{T_s}\right)$: Figure 2 : Table I

$\hat{\mathbf{D}}^C_k$ passthrough mat., known PWA ex. controller $= b\left(K_p + \frac{K_d}{T_s}\right)$: Eq. (60) : Table I

D^P passthrough mat., unknown true LTI plant model $= 0$: Figure 2 : Table I

$\hat{D}^P$ passthrough matrix of the known LTI plant model $= 0$: Section V-A : Table I

$\mathscr{E}$ edge effect filt., $I_{N-\mu_g+1 \times N-\mu_g+1}$ with first/last 35 diag. elements zeroed : Eq. (64) : Appendix B

e_k measured error in the example printhead system $= r_k - (y^P_k + \omega_y) \in \mathbb{R}$: Figure 2

e_{switch} threshold e_k magnitude for switching in example controller : $= 2\,\text{mm}$: Eq. (60) : Table I

$\mathbf{F}_k$ active affine state bias of PWA sys. at time k $= \sum_{q=1}^{|Q|} F_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_x}$: Eq. (2-5)

$F_{q,k}$ affine state bias of location q at time k $\in \mathbb{R}^{n_x}$: Eq. (1) : Definition 1

$\overline{\mathbf{F}}_k$ active affine state bias of a PWA inv. at time k $\in \mathbb{R}^{n_x}$: Eq. (14), (28) : Theorems 1 & 2

$\tilde{\mathbf{F}}^s_k$ active affine state bias of an inverse's stable modes $= \mathscr{I}^s V \overline{\mathbf{F}}_k \in \mathbb{R}^{n_s}$: Eq. (53)

$\tilde{\mathbf{F}}^u_k$ active affine state bias of inverse's unstable modes $= \mathscr{I}^u V \overline{\mathbf{F}}_k \in \mathbb{R}^{n_u}$: Eq. (53)

$\tilde{F}^u_{q,k}$ affine state bias of loc. q of inv.'s unstable modes $\in \mathbb{R}^{n_u}$: Eq. (59)

$\hat{\mathbf{F}}_k$ affine state bias of known monolithic example $= \begin{bmatrix} \hat{B}^P \hat{\mathbf{D}}^C_k \\ \hat{\mathbf{B}}^C_k \end{bmatrix} r_k$: Eq. (62) : Table I

$\mathscr{F}$ lifted lowpass filter matrix derived from $C^{LP}(z)$ $\in \mathbb{R}^{N-\mu_g+1 \times N-\mu_g+1}$: Eq. (77) : Appendix B

$\mathbf{G}_k$ active affine output bias of PWA sys. at time k $= \sum_{q=1}^{|Q|} G_{q,k} K_q(\delta_k) \in \mathbb{R}^{n_y}$: Eq. (2-5)

$G_{q,k}$ affine output bias of location q at time k $\in \mathbb{R}^{n_y}$: Eq. (1) : Definition 1

$\overline{\mathbf{G}}_k$ active affine output bias of PWA inv. at time k $\in \mathbb{R}^{n_y}$: Eq. (14), (28) : Theorems 1 & 2

$\mathcal{G}_k$ affine output bias of $\mu_g \geq 1$ PWA output preview $= \mathbf{C}_{k+\mu_g} \sum_{s=0}^{\mu_g-1} \left(\left(\prod_{m=s+1}^{\mu_g-1} \mathbf{A}_{k+m} \right) \mathbf{F}_{k+s} \right) + \mathbf{G}_{k+\mu_g}$ Eq. (16) : Lemma 2

$\hat{\mathbf{G}}_k$ affine output bias of known monolithic example $= 0$: Eq. (62) : Table I

$\hat{\mathbf{g}}$ lifted system input-output model (from $\mathbf{u}$ to $\hat{\mathbf{y}}$) $\in \mathbb{R}^{N-\mu_g+1} \to \mathbb{R}^{N-\mu_g+1}$: Eq. (71)

$\hat{\mathbf{g}}^i$ function from the lifted input to the i^{th} element of the lifted output : $\in \mathbb{R}^{N-\mu_g+1} \to \mathbb{R}$: Eq. (72)

$\hat{\mathbf{g}}^{-1}$ lifted input-output model inverse (from $\hat{\mathbf{y}}$ to $\mathbf{u}$) $\in \mathbb{R}^{N-\mu_g+1} \to \mathbb{R}^{N-\mu_g+1}$: Eq. (69) : Section V-B

$H(\bullet)$ Heaviside step function evaluated element-wise $\in \mathbb{R}^{n_P} \to \mathbb{B}^{n_P}$: Eq. (5)

$I_{\bullet\times\bullet}$ square identity matrix of dimension $\bullet$: Eq. (54)

$\not{I}$ exchange matrix [7] : Eq. (77)

$\mathcal{I}^{s}$ matrix extracting stable modes from inverse state $= \begin{bmatrix} I_{n_s \times n_s} & 0_{n_s \times n_u} \end{bmatrix} \in \mathbb{R}^{n_s \times n_x}$: Eq. (54)

$\mathcal{I}^{u}$ matrix extracting unstable modes from inv. state $= \begin{bmatrix} 0_{n_u \times n_s} & I_{n_u \times n_u} \end{bmatrix} \in \mathbb{R}^{n_u \times n_x}$: Eq. (54)

K_d derivative gain of example system's PD controller $= 3$: Eq. (60,76) : Table I

K_p switching proportional gain of example system's PD controller : $\in \mathbb{R}_{>0}$: Eq. (60,76) : Table I

$K_{p,1}$ low-error P-gain of example sys.'s PD controller $= 40$: Eq. (60) : Table I

$K_{p,2}$ high-error P-gain of example sys.'s PD controller $= 160$: Eq. (60) : Table I

$K_q(\bullet)$ selector function for the q^{th} location $= 0^{\prod_{i=1}^{|\Delta_q^*|} \left\| \delta_{q,i}^* - \bullet \right\|} \in \mathbb{B}^{n_P} \to \mathbb{B}$: Eq. (4)

k time step index $\in \mathbb{Z}$: Eq. (1) : Definition 1

ℓ ILC trial/iteration index $\in \mathbb{Z}_{\geq 0}$: Eq. (64)

L_ℓ the learning matrix in an ILC law at trial ℓ $\in \mathbb{R}^{N-\mu_g+1 \times N-\mu_g+1}$: Eq. (64)

N # of time increments in a finite state trajectory (# of samples minus 1) : $\in \mathbb{Z}_{>\mu_g}$: Section IV-B

NRMSE Normalized root mean square error $= \frac{\text{RMS}_{k \in [\![\mu_g, N]\!]}(e_k)}{\max_{k \in [\![\mu_g, N]\!]}(|r_k|)} = \frac{1}{\|\mathbf{r}\|_\infty} \frac{\|\mathbf{r}-\mathbf{y}\|_2}{\sqrt{N-\mu_g+1}}$: Eq. (74)

n_P # of hyperplanes partitioning the state space into convex polytopes : $\in \mathbb{Z}_{>0}$: Eq. (4)

n_s # of stable modes of a PWA system's inverse $\in \mathbb{Z}_{>0}$: Eq. (49) : Assumption (A9)

n_u # of unstable modes of a PWA system's inverse $\in \mathbb{Z}_{>0}$: Eq. (49) : Assumption (A9)

n_u # of system inputs / input space dimension $\in \mathbb{Z}_{>0}$: Definition 1

n_x # of system states / state space dimension $\in \mathbb{Z}_{>0}$: Definition 1

$n_{\hat{x}^P}$ # of states in the known printhead plant model $= 4$: Section V-A : Table I

n_y # of system outputs / output space dimension $\in \mathbb{Z}_{>0}$: Definition 1

P matrix whose each row is a hyperplane orientation vector [8, ch. 3] : $\in \mathbb{R}^{n_P \times n_x}$: Eq. (5)

P_o hyperplane orientation vectors in output space $P = P_o \mathbf{C}_k$: Assumption (A6)

$\tilde{P}^{s}$ hyperplane orientations of inverse's stable modes $PV^{-1} = \begin{bmatrix} \tilde{P}^{s} & 0_{n_P \times n_u} \end{bmatrix}$: Eq. (51) : Assumption (A10)

$\tilde{P}^{u}$ hyprpln. orientations of inverse's unstable modes $PV^{-1} = \begin{bmatrix} 0_{n_P \times n_s} & \tilde{P}^{u} \end{bmatrix}$: Eq. (52) : Assumption (A10)

$\hat{P}$ hyprpln. orientations, example monolithic model $= \begin{bmatrix} 0_{1 \times n_{\hat{x}^P} + n_{\hat{x}^C} - 1} & -1 \\ 0_{1 \times n_{\hat{x}^P} + n_{\hat{x}^C} - 1} & 1 \end{bmatrix}$: Eq. (63)

$\text{Pre}(\bullet)$ mapping from a set of states $\bullet$ to the "predecessor set" of states with next-time-step values $\in \bullet$ Eq. (52) : Assumption (A10b)

Q set of all locations in the PWA system state space partitioning : Definition 1

Q_q q^{th} location of a PWA system; it is the union of a set of disjoint convex polytopes $\subseteq \mathbb{R}^{n_x}$: Eq. (1) : Definition 1

$\mathcal{Q}$ zero phase shift lowpass filter matrix $\in \mathbb{R}^{N-\mu_g+1 \times N-\mu_g+1}$: Eq. (64,77)

q index of a location in a PWA system $\in [\![1, |Q|]\!]$: Eq. (1) : Definition 1

$\mathbb{R}$ domain of real numbers [9]

r_k output reference at time k $\in \mathbb{R}^{n_y}$: Definition 3

$\mathbf{r}$ lifted (i.e. time series) reference vector $= \begin{bmatrix} r_{\mu_g} & r_{\mu_g+1} & \cdots & r_N \end{bmatrix}^T \in \mathbb{R}^{N-\mu_g+1}$: Eq. (67)

$\sup_\circ \bullet$ supremum of $\bullet$ over $\circ$ Eq. (47) : Assumption (A8)

T_s sample period : $\in \mathbb{R}_{>0}$: Eq. (60) : Table I

u_k input vector at time k $\in \mathbb{R}^{n_u}$: Eq. (1) : Definition 1

$\mathbf{u}$ lifted input vector (finite time series trajectory) $= \begin{bmatrix} u_0 & u_1 & \cdots & u_{N-\mu_g} \end{bmatrix}^T \in \mathbb{R}^{N-\mu_g+1}$: Eq. (65)

$\mathbf{u}_\ell$ lifted input vector $\mathbf{u}$ value for ILC trial ℓ $\in \mathbb{R}^{N-\mu_g+1}$: Eq. (64)

V similarity transform matrix for decoupling modes $\in \mathbb{R}^{n_x \times n_x}$: Eq. (49) : Assumption (A9)

X_0 set of x_0 values from which all locations are reachable in finite time : Assumption (A1)

$\mathcal{X}^u$ set containing at least all solution values of $\tilde{x}^u_{k+1}$ $\subseteq \mathbb{R}^{n_u}$: Eq. (52) : Assumption (A10b)

x_k state vector at time step k $\in \mathbb{R}^{n_x}$: Eq. (1) : Definition 1

$\tilde{x}^s_k$ state of the PWA system inverse's stable modes $= \mathcal{J}^s V x_k \in \mathbb{R}^{n_s}$: Eq. (53)

$\tilde{x}^u_k$ state of the PWA system inverse's unstable modes $= \mathcal{J}^u V x_k \in \mathbb{R}^{n_u}$: Eq. (53)

$\hat{x}_k$ state of known example printhead system's monolithic model : $= \left[\left(\hat{x}^P_k\right)^T \quad \left(\hat{x}^C_k\right)^T \right]^T$: Eq. (62)

$\hat{x}^C_k$ state of known example feedback controller model $\in \mathbb{R}^{n_{\hat{x}^C}}$: Eq. (60)

$\hat{x}^P_k$ state of the known example plant model, a printhead positioning system : $\in \mathbb{R}^{n_{\hat{x}^P}}$: Section V-A

y_k output vector at time k $\in \mathbb{R}^{n_y}$: Eq. (1) : Definition 1

$\hat{y}_k$ modeled/predicted output vector at time k $\in \mathbb{R}^{n_y}$: Eq. (69)

$\mathbf{y}$ lifted (i.e. time series) measured output vector $= \begin{bmatrix} y_{\mu_g} & y_{\mu_g+1} & \cdots & y_N \end{bmatrix}^T \in \mathbb{R}^{N-\mu_g+1}$: Eq. (66)

$\hat{\mathbf{y}}$ lifted (i.e. time series) modeled/predicted output $= \begin{bmatrix} \hat{y}_{\ell,\mu_g} & \hat{y}_{\ell,\mu_g+1} & \cdots & \hat{y}_{\ell,N} \end{bmatrix}^T$: Section V-B

$\hat{\mathbf{y}}^i$ i^{th} element of lifted modeled/predicted output $\hat{\mathbf{y}}^i = \hat{\mathbf{g}}^i(\mathbf{u}) = \hat{y}_{\mu_g+i-1} \in \mathbb{R}$: Eq. (72)

$\mathbf{y}_\ell$ lifted measured output vector $\mathbf{y}$ for ILC trial ℓ $\in \mathbb{R}^{N-\mu_g+1}$: Eq. (64)

y^C_k feedback controller output, part of total motor voltage : $\in \mathbb{R}$: Figure 2 : Section V-A

y^P_k plant output, printhead position along guide rail $\in \mathbb{R}$: Figure 2 : Section V-A

$\hat{y}^P_k$ modeled/predicted plant output, printhead position along guide rail : $\in \mathbb{R}$: Eq. (61b)

$\mathbb{Z}$ domain of integers [10]

z independent variable in the z-domain [11] : $\in \mathbb{C}$

β vector of hyperplane offsets [8, ch. 3] $\in \mathbb{R}^{n_P}$: Eq. (5)

β_o offsets for hyperplanes in the output space $\beta = \beta_o - P_o \mathbf{G}_k \in \mathbb{R}^{n_P}$: Assumption (A6)

$\hat{\beta}$ hyprpln. offsets, example monolithic model $= \begin{bmatrix} -e_{\text{switch}} & -e_{\text{switch}} \end{bmatrix}^T$: Eq. (63)

γ_∇ learning gain of gradient ILC : $\in \mathbb{R}_{>0}$: Eq. (70)

γ_P learning gain of P-type ILC : $\in \mathbb{R}_{>0}$: Eq. (73)

Δ^*_q set of the signature of each polytope in Q_q $= \{\delta^*_{q,1}, \delta^*_{q,2}, \cdots\}$: Eq. (4)

$\hat{\Delta}^*_1$, $\hat{\Delta}^*_2$ polytope signature sets for both locations of the example monolithic closed-loop printer model $= \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, respectively : Eq. (63)

δ_k localization vector indicating on which side of each hyperplane (i.e. in which polytope) x_k lies $= \delta(x_k) = H(Px_k - \beta) \in \mathbb{B}^{n_P}$: Eq. (5) : [8, ch. 3]

$\hat{\delta}_k$ localization vec., known monolithic printer model $\in \mathbb{B}^{n_P}$: Section V-B

$\delta^*_{q,i}$ unique signature of the i^{th} polytope in Q_q $\in \mathbb{B}^{n_P}$: Eq. (4)

ε "small" number used in epsilon-delta definitions $\in \mathbb{R}_{>0}$: Eq. (48) : Assumption (A8)

η_1, η_2 receding negative, positive (respectively) points in time : $\in \mathbb{Z}$: Eq. (48) : Assumption (A8)

μ_c common component relative degree shared by all locations under Assumption (A4) : $\in \mathbb{Z}$

μ_g global dynamical relative degree of a PWA sys. $\in \mathbb{Z}$: Definition 2

μ_q relative degree the q^{th} location in a PWA sys. $\in \mathbb{Z}$: Definition 1

χ^s_k state, unforced version of PWA inverse's stable modes : $\in \mathbb{R}^{n_s}$: Eq. (50) : Assumption (A9)

χ^u_k state, unforced version of PWA inverse's unstable modes : $\in \mathbb{R}^{n_u}$: Eq. (50) : Assumption (A9)

$\Psi_k(\cdots)$ input preview term of PWA output preview $= \mathbf{C}_{k+\mu_g} \sum_{s=1}^{\mu_g-1} \left(\left(\prod_{m=s+1}^{\mu_g-1} \mathbf{A}_{k+m} \right) \mathbf{B}_{k+s} u_{k+s} \right)$ Eq. (16) : Lemma 2

ω_c process noise, random sample of a Gaussian distribution : $\in \mathbb{R}$: Figure 2 : Section V-A

ω_y measurement noise, random sample of a Gaussian distribution : $\in \mathbb{R}$: Figure 2 : Section V-A

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